BIR JINSLI BO'LMAGAN IMMIGRATSIYALI TARMOQLANUVCHI TASODIFIY JARAYONLARNING ASIMPTOTIK XOSSALARI

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Annotatsiya. Kamayuvchi immigratsiyali, holatga bog'liq immigratsiyali tarmoqlanuvchi tasodifiy jarayonlar, bir jinsli bo'lmagan immigratsiyali Galton-Vatson tarmoqlanuvchi jarayonilari va ularning asimptotik xossalari o'rganilgan . Bir jinsli bo'lmagan immigratsiyali kritik tarmoqlanuvchi tasodifiy jarayoni uchun limit teorema isbotlangan.

Kalit so'zlar: Kamayuvchi immigratsiya, Galton-Vatson tarmoqlanuvchi jarayonilari, demografiya, meditsina, molekulyar biologiya, yadro fizikasi,

Kamayuvchi immigratsiyali Galton-Vatson tarmoqlanuvchi tasodifiy jarayonning asimptotik hossasi o'rganilgan. Qaralayotgan Galton-Vatson tarmoqlanuvchi jarayoni kiritik jarayon, hamda kamayuvchi immigratsiyali jarayon deb faraz qilingan. Ushbu farazlarda bunday jarayon sonli xarakteristikalari uchun limit teorema isbotlaymiz. Holatga bog'liq bo'lgan immigratsiyali Galton-Vatson tarmoqlanuvchi tasodifiy jarayonini ko'rib chiqamiz. Aytaylik, μ_n Galton-Vatson jarayonining n- vaqt momentidagi zarralar soni bo'lsin (n = 1,2,...,μ0 = 1).

Faraz qilaylik quyidagi shartlar bajarilgan bo'lsin:

$$F'(1) = 1, \qquad 0 < F''(1) = 2b < \infty \qquad (1)$$

$$\max_n \alpha_n < \infty, \qquad \max_n \beta_n < \infty \qquad (2)$$

$$0 < \alpha_n \to 0, \qquad \beta_n \to 0, \qquad n \to \infty \qquad (3)$$

Lemma Holatga bog'liq immigratsiyali tarmoqlanuvchi jarayonning hosil qilish funksiyasi uchun quyidagi munosabatlar o'rinli:

$$\Phi_n(x)=1-\sum_{k=1}^{m}\left[\sum_{s=0}^{n-1}\left(1-g_{kn}\left(F_s(x)\right)F^k{}_{s+1}(x)p_{0k}(n-s-1)\right)\right] \quad (4)$$

$$F^k{}_s(x)=1-\frac{k}{\frac{1}{1-x}+sB}+o\left(\frac{1}{s}\right), \qquad s\to\infty$$

bu yerda $F_0(x)=x,\ F_1(x)=F(x)\ F_{n+1}(x)=F\left(F_n(x)\right),\ n=0,1,2,...$

$$\Phi_n(x)=1-\sum_{k=1}^{m}\left[\sum_{s=0}^{n-1}\left(1-g_{kn}\left(F_s(x)\right)F^k{}_{s+1}(x)p_{0k}(n-s-1)\right)\right]$$

$$g_{k,n}(x)=\sum_{j=0}^{\infty}q_{kj}(n)x^j, \quad |x|\le 1, \quad k=0,1,...,m,\ q_{kj}(\mathrm{n})\ge 0,$$

$$\sum_{j=0}^{\infty}q_{kj}(n)=1, \qquad n=0,1,2,... \quad .$$

$$F^k{}_s(x)=1-\frac{k}{\frac{1}{1-x}+sB}+o\left(\frac{1}{s}\right), \qquad F^k{}_s(1)=1+o\left(\frac{1}{s}\right),$$

$$F^k{}_{s+1}(x)=1-\frac{k}{\frac{1}{1-x}+(s+1)B}+o\left(\frac{1}{s}\right),$$

$$F_{s+1}(x)=1-\frac{1}{\frac{1}{1-x}+(s+1)B}+o\left(\frac{1}{s}\right), \qquad F'{}_{s+1}(x)=\frac{1}{(1+(s+1)B(1-x))^2}$$

$$F'{}_{s+1}(1)=1$$

$$F^{k-1}{}_{s+1}(x)=1-\frac{k-1}{\frac{1}{1-x}+(s+1)B}+o\left(\frac{1}{s}\right), \quad F^{k-1}{}_{s+1}(1)=1+o\left(\frac{1}{s}\right),$$

$$F_s(x)=1-\frac{1}{\frac{1}{1-x}+sB}+o\left(\frac{1}{s}\right), \quad F_s(1)=1+o\left(\frac{1}{s}\right),$$

$$F'_s(x)=\frac{1}{(1+sB(1-x))^2} \qquad F'_s(1)=1$$

$$g_{kn}(F_s(x))=\sum_{j=0}^{\infty}q_{kj}(n)(F_s(x))^j \ ,$$

$$g'_{kn}(F_s(1))=g'_{kn}\left(1+o\left(\frac{1}{s}\right)\right)=\sum_{j=0}^{\infty}q_{kj}(n)\,\mathrm{j}\left(1+o\left(\frac{1}{s}\right)\right)^{j-1}=\alpha_{kn}$$

$$\alpha_n=\max_{0\le k\le m}g^1_{kn}(1) \qquad \sum_{k=1}^{m}\alpha_{kn}\le m\alpha_n$$

$$\Phi'_n(x)=\sum_{k=1}^{m}\left[\sum_{s=0}^{n-1}\left(\sum_{j=0}^{\infty}q_{kj}F'_s(x)j(F_s(x))^{j-1}kF^{k-1}_{s+1}(x)F'_{s+1}(x)p_{0k}(n-s-1)\right)\right]$$

$$\Phi'_n(1)=\sum_{k=1}^{m}\left[\sum_{s=0}^{n-1}\left(\sum_{j=0}^{\infty}q_{kj}F'_s(1)j(F_s(1))^{j-1}kF^{k-1}_{s+1}(1)F'_{s+1}(1)p_{0k}(n-s-1)\right)\right]=$$

$$=\sum_{k=1}^{m}\left[\sum_{s=0}^{n-1}\left(\sum_{j=0}^{\infty}q_{kj}j\left(1+o\left(\frac{1}{s}\right)\right)^{j-1}k(1+o\left(\frac{1}{s}\right))p_{0k}(n-s-1)\right)\right]=$$

$$=\sum_{k=1}^{m}\left[\sum_{s=0}^{n-1}\left(\sum_{j=0}^{\infty}q_{kj}j\left(1+o\left(\frac{1}{s}\right)\right)^{j}kp_{0k}(n-s-1)\right)\right]=\sum_{k=1}^{m}\left[\sum_{s=0}^{n-1}\left(\alpha_{kn}kp_{0k}(n-s-1)\right)\right]\le$$

$$\le\sum_{s=0}^{n-1}\alpha_n m\left[\sum_{k=1}^{m}\left(kp_{0k}(n-s-1)\right)\right]=\sum_{s=0}^{n-1}\alpha_n P_{0j}$$

(4) dan x ga ko’ra xosila olib, x ga 1 qo’yganda jarayon momentlari uchun

$$A_{n+1}=\sum_{j=0}^{n}\alpha_j P_{0j}(\mathrm{n}) \qquad (5)$$

$$B_{n+1}=2b\sum_{j=0}^{n}\alpha_j P_{0j}(n-j)+\beta_j P_{0j}(n) \qquad (6)$$

ga ega bo’lamiz.

Shu bilan birga α_n ning nolga yaqinlashish tezligi $\alpha_n \sim n^{-1}L(n),\ n\to\infty$ bo’lsin, bu yerda L(n) cheksizlikda sekin o’zgaruvchi funksiya.

$L^*(n)=\sum_{k=1}^{n}\alpha_k \sim \sum_{k=1}^{n}\frac{L(k)}{k}$, $n\to\infty$ funksiya ham cheksizlikda sekin o'zgaruvchi funksiya bo'ladi.

Teorema. Agar $\beta_n = o(\alpha_n \ln n)$ va $L^*(n)\to\infty$, $n\to\infty$, bo'lsa

u holda $n\to\infty$ da $A_n \sim L^*(n)$, $B_n \sim 2bnL^*(n)$.

Teoremaning isboti. Bizga ma'lumki, $P_{0j}\to 1$ da (5) dan

$$A_n=\sum_{j=0}^{n}P_{0j}\alpha_j \sim \sum_{j=0}^{n}\alpha_j \sim L^*(n). \quad n\to\infty.$$

Ushbu munosabat teoremaning birinchi qismi o'rinli ekanligini ko'rsatadi.

Endi sekin o'zgaruvchi funksiyaning hossalaridan foydalansak

$\lim L(n)/L^*(n)=0$ $n\to\infty$ va (6) tenglikka ko'ra

$$B_n = O(L(n)) + 2bnL^*(n)(1+o(1)) - 2bnL(n)(1+o(1)) \sim 2bnL^*(n) \quad n\to\infty$$

Yuqoridagi munosabatlardan teoremaning ikkinchi tasdig'i ham o'rinli ekanligi kelib chiqadi.

Maqolamizda orqali bir nechta holatga bog'liq immigratsiyali tarmoqlanuvchi tasodifiy jarayonlar o'rganilgan. Jumladan, kamayuvchi immigratsiyali Galton-Vatson tarmoqlanuvchi tasodifiy jarayonning asimptotik hossasi o'rganilgan. Qaralayotgan Galton-Vatson tarmoqlanuvchi jarayoni kiritik jarayon, hamda kamayuvchi immigratsiyali jarayon deb faraz qilingan. Ushbu farazlarda bunday jarayon sonli xarakteristikalari uchun limit teorema isbotlangan.

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