QO‘ZG‘ALISHINING RANGI UCHGA TENG BO’LGAN UMUMLASHGAN FRIDRIXS MODELI XOS QIYMATLARI SONI VA O‘RNI

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$\mathbb{T}^3 = \mathbb{T}^1 \times \mathbb{T}^1 \times \mathbb{T}^1$ uch o'lchamli tor bo'lsin. $L^2(\mathbb{T}^3)$ orqali $\mathbb{T}^3$ da aniqlangan modulining kvadrati bilan integrallanuvchi funksiyalarning Hilbert fazosini belgilaymiz.

$C^{(1)}(\mathbb{T}^1)$ orqali $\mathbb{T}^1$ (mos ravishda $\mathbb{T}^3$) da aniqlangan, bir marta uzluksiz differensiallanuvchi, haqiqiy qiymatli funksiyalar to’plamini belgilaymiz.

Qo’zg’alishining rangi uchga teng bo’lgan umumlashgan Friedrixs modeli $L^2(\mathbb{T}^3)$ fazoda quyidagicha aniqlanadi:

$$H_{\mu_1,\mu_2,\mu_3}(p) = H_0(p) + V, \qquad (3.1.1)$$

$$V = \mu_1 V_1 + \mu_2 V_2 + \mu_3 V_3\,,\ \mu_1,\mu_2,\mu_3 > 0,$$

Bunda $H_0(p)\,,\ p \in \mathbb{T}^3$ operator $\omega_p(\cdot) \coloneqq \omega(\,p,\ \cdot\,)$ funksiyaga ko'paytirish operatori:

$$(H_0(p)f)(q) = \omega_p(q)f(q)\,, \quad f \in L^2(\mathbb{T}^3)$$

va

$$(V_i f)(q) = \varphi_i(q_i) \int_{\mathbb{T}^3} \varphi_i(s_i) f(s) ds\,, \qquad i = 1,2,3.$$

Faraz 1. Quyidagi shartlar o’rinli bo’lsin deylik:

(i) $\varphi_i(\cdot),\ i = 1,2,3$ funksiya $C^{(1)}(\mathbb{T}^1)$ to'plamga tegishli bo'lsin

(ii) $\omega(\cdot\,,\cdot)$ funksiya $(\mathbb{T}^3)^2 = \mathbb{T}^3 \times \mathbb{T}^3$ da aniqlangan haqiqiy analitik va $(p_0, q_0) \in (\mathbb{T}^3)^2$ nuqtada yagona aynimagan maxsimumga nuqtaga bo’lsin.

V_i qo`zg`alish operatorlari chekli o`lchamli opertorlar ekanligidan ular kompakt operatorlar bo`ladi va Weyl teoremasiga ko`ra $H_{\mu_1,\mu_2,\mu_3}(p)$ operatorning muhim spektri uchun quyidagi tenglik o`rinli bo`ladi:

$$\sigma_{ess}\left(H_{\mu_1,\mu_2,\mu_3}(p)\right) = \sigma_{ess}\big(H_0(p)\big) = \sigma\big(H_0(p)\big) = [m(p), M(p)]\,,$$

$$m(p) = \min_{q\in\mathbb{T}^3} \omega_p(q)\,, \qquad M(p) = \max_{q\in\mathbb{T}^3} \omega_p(q)\,.$$

Faraz 2. Har bir $z>M(p)$ uchun quyidagi munosabat o`rinli bo`lsin:

$$\int_{\mathbb{T}^3} \frac{\varphi_i(s_i)\varphi_j(s_j)}{\omega_p(s)-z} ds = 0\,, \quad i \neq j, \quad i,j = 1,2,3.$$

Ta'kidalab o`tamizki, Faraz 1 ga ko`ra $p = p_0 \in \mathbb{T}^3$ nuqtaning shunday δ – atrofi $U_\delta(0) \subset \mathbb{T}^3$ va unda aniqlangan $q_0 : U_\delta(0) \subset \mathbb{T}^3$ analitik funksiya mavjudki har bir $p \in U_\delta(0)$ uchun $q_0(p) = \left(q_1^0(p), q_2^0(p), q_3^0(p)\right) \in \mathbb{T}^3$ nuqta $\omega_p(\cdot)$ funksiyaning yagona aynimagam maksimum nuqtasi bo`ladi. Bundan tashqari $\varphi_i\left(q_i^0(p)\right) = 0\,,\ \ i = 1,2,3\,,\ \ p \in U_\delta(0)$ shart bajarilganda quyidagi

$$\int_{\mathbb{T}^3} \frac{\varphi_i^2(s_i)}{\omega_p(s)-m(p)} ds > 0$$

integral mavjud bo`ladi.

Ta'rif 1. $\varphi_i\left(q_i^0(p)\right) = 0$ bo`lganda $\mu_i(p) > 0$ sonini quyidagicha aniqlaymiz

$$\mu_i(p) = \left(\int_{\mathbb{T}^3} \frac{\varphi_i^2(s_i)ds}{\omega_p(s)-M(p)} \right)^{-1} < 0\ ,$$

va agar $\varphi_i\big(q_0(p)\big) \neq 0,$ bo`lsa $\mu_i(p) = 0$ deb qabul qilamiz. Bunda $i = 1,2,3,\ \ p \in U_p(0)$.

Teorema

I. $H_{\mu_1,\mu_2,\mu_3}(p)$ operator $M(p)$ dan yuqorida yagona $E_i(\mu_i\,, p)$ xos qiymatga ega bo`lishi uchun $\mu_i > \mu_i(p)\,,\ 0 < \mu_j < \mu_j(p)\,,\ 0 < \mu_k < \mu_k(p)$ $k = 1,2,3$ $i \neq j$ $i \neq k$ munosabatning bajarilishi zaraur va yetarlidir. $E_i(\mu_i\,, p)$ funksiya $\mu_i$ bo`yicha $(\mu_i(p)\,, +\infty)$ da monoton kamayuvchi haqiqiy analitik hamda p bo`yicha $U_p(0)$ da haqiqiy analitik. Xos qiymaga mos xos funkiya quyidagi ko`rinishda

$$\psi_i\big(\mu_i\,, p\,, \cdot\,, E_i(\mu_i\,, p)\big) = \frac{C_i\mu_i\varphi_i(q_i)}{\omega_p(q)-E_i(\mu_i\,, p)} \qquad (*)$$

hamda $C^1(\mathbb{T}^3)$ to`plamga tegishli bo`ladi bunda $C_i \neq 0$ normal ko`paytuvchi. Bundan tashqari quyidagi

$$\psi_i : U_p(0) \to L^2(\mathbb{T}^3), \quad p \to \psi_i(\mu_i, p, \cdot, E_i(\mu_i, p))$$

va

$$\psi_i : (\mu_i(p), +\infty) \to L^2(\mathbb{T}^3), \quad \mu_i \to \psi_i(\mu_i, p, \cdot, E_i(\mu_i, p))$$

Akslantirishlar vektor qiymatli haqiqiy va mos ravishda $U_p(0)$ va $(\mu_i(p), +\infty)$ da analitik bo`ladi.

II. $H_{\mu_1,\mu_2,\mu_3}(p)$ operator $m(p)$ dan quyida uchta $E_i(\mu_i, p)$, $i = 1,2,3$ xos qiymatlarga ega bo`lishi uchun $\mu_i > \mu_i(p)$, $i = 1,2,3$ bo`lishi zarur va yetarli. Ularga mos xos funksiyalar (*) kabi aniqlanadi.

FOYDALANILGAN ADABIYOTLAR RO‘YXATI:

1. S. N. Lakaev, Sh. Kh. Kurbanov and Sh. U. Alladustov, "Convergent expansions of eigenvalues of the generalized Friedrichs model with a rank-one perturbation," Complex Analysis and Operator Theory

2. Sh. Kh. Kurbanov and S. T. Dustov, "Puiseux Series Expansion for Eigenvalue of the Generalized Friedrichs Model with the Perturbation of Rank One," Labochevskiy Journal of Mathematics